

CHERN SIMONS TOPOLOGICAL INTERACTIONS ACROSS DIMENSIONS

► Chern-Simons interactions in 5d

$$\mathcal{L}_{CS} = \frac{N_c \epsilon^{ABCDE}}{24\pi^2} \text{Tr} \left(A_A \partial_B A_C \partial_D A_E - \frac{3i}{2} A_A A_B A_C \partial_D A_E - \frac{3}{5} A_A A_B A_C A_D A_E \right)$$

encode holographically physical (safe, flavor-chiral) anomalies of our familiar world: the 4d Standard Model and its effective gauged chiral models [C T Hill & Zachos, Ann Phys **323** (2008) 3065-3073]. May also lead to new dimensional deconstruction-based phenomena **accessible at colliders**.

⊗ However, they **also** model, in a remarkable **3d superconformal** lagrangian model (Bagger-Lambert-Gustavsson) with a finite number of fields, the dual to two M2 branes superposed on an orbifold R^8/Z_2 in M-theory—the strong-coupling limit of **quantum gravity**.

⌋ This **CS interaction is predicated on Ternary algebraic structures**, Nambu Brackets under a very stringent condition (FI). Finite-dimensional exemplars of such algebras satisfying FI are provably **few** (essentially **one**).

★ Still, a **New infinite dimensional ternary algebra** was discovered, based on the Witt algebra, in [Curtright, Fairlie, & Zachos, Phys Lett **B666** (2008) 386-390],

$$[Q_k, Q_m, Q_n] = (k - m)(m - n)(k - n) R_{k+m+n} ,$$

$$[Q_p, Q_q, R_k] = (p - q) \left(Q_{k+p+q} + s k R_{k+p+q} \right) ,$$

$$[Q_p, R_q, R_k] = (k - q) R_{k+p+q} , \quad [R_p, R_q, R_k] = 0 ,$$

with s a parameter. Only for $s = \pm 2i$ is the FI satisfied.

○ This intriguing compliance with FI was finally **fully understood** in an unexpected isomorphic correspondence with Nambu's Classical Brackets (**unrelated**, in principle!) in [Curtright, Jin, Mezincescu, Fairlie, & Zachos, Phys Lett **B675** (2009) 387-392].

■ These results were further extended, heuristically, to cover several other concrete cases: in fact, **all known** FI-compliant Ternary Algebras.
 ∼ Applications to new lagrangian M2-model building, and consistency checks?

⊙ **Spacetime umbral transforms of solitons** invented in [Zachos, Int J Mod Phys **A23** (2008) 2005-2014], were finally appreciated to be **dispersive**, in contrast to their continuum correspondents (unpublished).

✓[Curtright & Zachos, arXiv:0909.2424]: Evolution profile holographic interpolation dynamics utilizing **Schröder's functional equation**,

$$\lambda \Psi(x) = \Psi(f(x)).$$

Dynamical systems, Renormalization Group relevance.

